WavePiston Project

Structural Concept Development
Hydrodynamic Aspects

Preliminary Analytical Estimates

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0	Script version	March 9, 2010	SSC	PMJ	JHA
No.	Revision	Date	Prepared	Checked	Approved

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1 Introduction

Many initiative exist in the wave energy device developments. However, it is still a young area in terms of long-term practical utilization. Evans (1981) and Falnes (2002) provides extensive descriptions on theoretical calculations of wave energy extraction. Recent detailed reviews by Falnes (2007) and Falcão (2009) also describe the development in the area of wave energy devices.

Herein, hydrodynamic analytical estimates of the WavePiston system to generate flat plate data are presented.
The energy contribution of non-linear wave forces on the moving plates of the WavePiston are considered for two non-linear waves and two plate sizes. First the wave theory used will be introduced and the method to solve the problem will be explained. In the end the results will be discussed, recommendation to further analysis will be given and some main concerns about the construction of the WavePiston is mentioned.
The present investigations have been conducted during four weeks, January, 2010.

2 Concept

The present concept, the WavePiston system, will shortly be described. First, a series of thin plates are mounted on a core circular cylinder, as shown on the unit in Fig. 1. Second, the system is to be located near the free surface. The system works as follows:

1. Water is sucked into a separate pipe mounted above the WavePiston pipe via a series of holes;
2. The twin plates moves. The plate velocity $\dot{x}_p$ is a function of the wave celerity, c, and therefore $\dot{x}_p \in [0; c]$;
3. The WavePiston pumps water into the core circular cylinder. The pumping rate and thus the water pressure in the core cylinder dictates the energy converted;
4. At the end of the core cylinder, at shore is located a turbine and power converter equipment.

Further descriptive details can be found in the brochure by von Bülow *et al.* (2009).

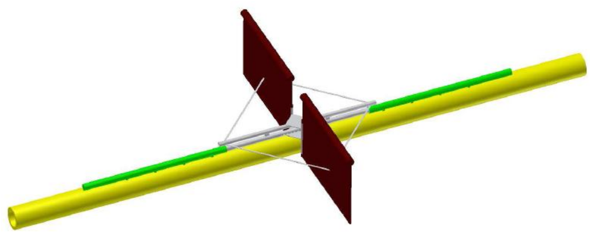

Figure 1: *Sketch of WavePiston unit.*

3 Method of Analysis

In this section is outlined the basic theory and assumptions from which the results presented are based on.
The problem to be solved is rather complex because the force acting on the plate and the displacement, velocity and acceleration of the plate are interdependent. For nonlinear waves the computational time is relative large because the time steps have to be quite small in order to capture the changes. It has therefore not been possible to investigate many plate sizes for different wave cases. In the report it is instead outlined how the power generated by a plate in a oscillating flow can be calculated. The problem is solved for two wave cases and two plate sizes.
In order to calculate the power absorbed by the device properly it is necessary with more investigations on how the plate moves relatively to the water and how the forces acts on the plate.

3.1 Key assumptions

- Regular waves;
- Current contribution is not taken into account;
- Incompressible and irrotational flow;
- Wave breaking effects are not considered;
- Plates are submerged at all time;
- The effect of the motion at the offshore boundary is not taken into account;
- Vortex shedding is not taken into account.

3.2 Wave theory

Classical wave theory is based on potential flow theory rooted in the Laplace equation. The boundary condition at the free-surface consist of dynamic and kinematic constraints. The dynamic condition is based on an unsteady Bernoulli equation. It is this condition which introduces the nonlinear physics of water waves.

Linear theory is used when small amplitude waves can be assumed. Otherwise higher order Stokes approximation or the stream function theory apply. The stream function theory has the advantage over most other theories as it can be applied usefully to all wave conditions. However, it should be noted that these classical wave theories have their limitations. Improvement can be found in the numerical method literature. The research in steep and breaking wave physics are still on-going.

3.3 Wave characteristics

The wave data used are with intermediate water depth characteristics ($\frac{1}{20} < \frac{h_0}{L} < \frac{1}{2}$) in mind as listed in Table 1. The mean water level is h_0, H_s the significant wave height, T the wave period and L denotes the wave length. Using the classical linear dispersion relationship, the wave frequencies are defined as

$$\omega_n = \sqrt{g\, k_n \tanh(k_n\, h_0)}\,, \tag{1}$$

where $k_n = n\,\pi/L$ is the wave number for $n = 0,\ 1,\ 2...$, and g is gravity due to acceleration. The corresponding first natural frequency of the wave $f_1 = \omega_1/2\pi$.

The group velocity, c_g represents a train of propagating waves and depends on the wave length (or wave period) due to the dispersion relation for water waves under the action of gravity. Therefore, the group velocity behaves differently in the limits of shallow, intermediate and deep water depths. The speed of the wave form, the wave celerity, $c = L/T$ (T is the regular wave period) and the speed of the wave group is defined as $c_g = m\,c$ where $m = 1/2$ for deep water and $m = 1$ for shallow depths, respectively. The group velocity at intermediate depth is:

$$c_g = \frac{1}{2}c\Big(1 + \frac{4\pi h_0}{L}\,\frac{1}{sinh(\frac{4\pi h_0}{L})}\Big)\,, \tag{2}$$

In order to simplify the calculations it is decided to keep the top elevation of the twin plates of the unit of the WavePiston constant instead of varying in time. It is therefore assumed that the twin plates is fully submerged at all times. The top elevation of the submerged plates, z_p^{top}, is therefore below the trough of the wave and is listed in Table 1 for two incident waves. This assumption entails that the force acting on the plate becomes smaller because the velocity and acceleration in the flow decreases with the depth. In the power-calculations on the other hand it is assumed that the plate contributes to the power generation over the whole wave period while the plate in the WavePiston-system only contributes to the power generation when the force has obtained a certain value and is then constant, as illustrated in Fig. 2. The smaller force which is generated because the top elevation of the plate is kept constant is therefore equalized by the fact that the plate in this calculation contributes to the power generation over the whole period.

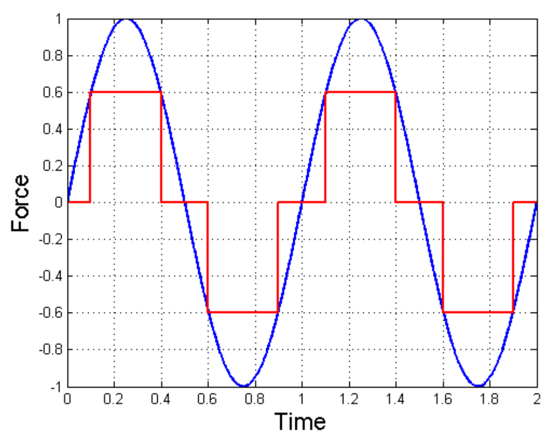

Figure 2: *Plate contribution to the power-generation. In this report it is assumed that the plate contribute over the whole wave period (−). In the WavePiston-system the plate only contributes to the power generation when the force has obtained a certain value and is then constant (−).*

Case #1 represents typical one year return wave conditions at Horns Rev, Denmark. The surface elevation, η, the horizontal and vertical velocity of the flow, u and w, at the surface, and the horizontal acceleration off the flow du/dt at the surface are shown in Fig. 3 for the two wave cases.

3.4 Wave forces

The in-line wave force per unit length for fixed structures can be described by the Morison equation, Morison *et al.* (1950):

$$F_x(U, \dot{U}) = \frac{1}{2}\rho C_D D U|U| + \rho C_M A \dot{U}\,, \tag{3}$$

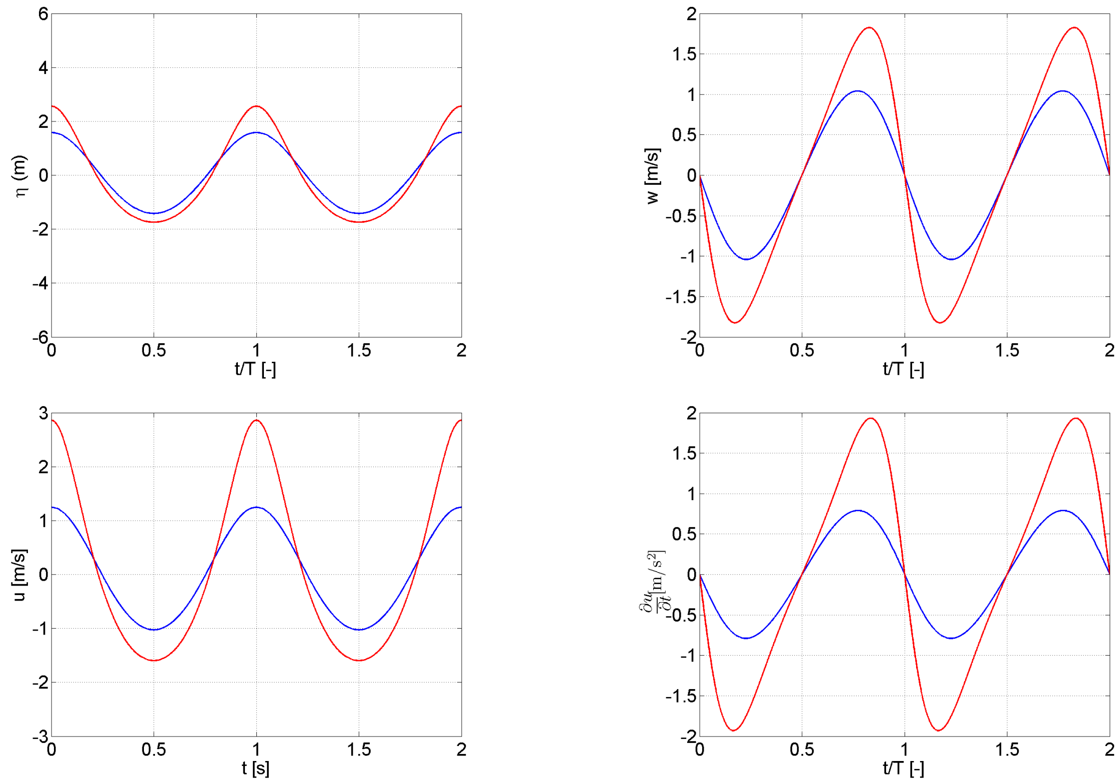

Figure 3: *Free-surface solution based on Stream Function theory.* — *Case #1,* — *Case #2.*

Table 1: *Wave data.*

Parameter		Case #1	Case #2	Parameter		Case #1	Case #2
h_0	[m]	14	30	h_0/L	[-]	0.19	0.26
H_s	[m]	4.3	3	η_{max}	[m]	2.56	1.58
T	[s]	7.4	9	$S = k\eta_{max}$	[-]	0.22	0.08
L	[m]	74.7	117.5	η_{trough}	[m]	-1.74	-1.42
f_1	[Hz]	0.13	0.11	z_p^{top}	[m]	-2.0	-1.5
c	[m/s]	10.1	13.1	l_{unit}	[m]	75	118
c_g	[m/s]	16	16.5	$Ur = \frac{H_s L^2}{h_0^3}$	[-]	8.7	1.5

where U is the free-stream velocity, ρ is the density of water, C_D is the drag coefficient and $C_M(= C_m+1)$ is the inertia coefficient. The first and second terms describe the drag and inertia contribution.

The in-line wave force for a moving body (herein plate) relative to the flow can be estimated based on a modification of Eqn. (3):

$$F_x(U,\dot{U},\dot{x}_s,\ddot{x}_s) = F_D + F_{HM} + F_{FK} = \frac{1}{2}\rho C_D D(U-\dot{x}_s)|U-\dot{x}_s| + \rho C_m A(\dot{U}-\ddot{x}_s) + \rho A\dot{U}\,, \quad (4)$$

where U is the velocity of the plate in the in-line direction. The first term represents the drag force while the second term describes the hydrodynamic mass force. The third term is due to the pressure gradient named the Froude-Krylov force. Eqn. (4) reduces to Eqn. (3) when $x_s = 0$. It is assumed that the optimum conditions is when the waves act perpendicular to the plates. In this situation, the coefficients: $C_D \approx 2$ and $C_m \approx 1$. Further, it can be observed from Eqn. (4) that the Froude-Krylov force contribution will become dominating with increasing velocity of the plate.

3.5 Plate characteristics

The twin plates moves primarily in-line with the wave motion generating wave forces F_x. The in-line force oscillates primarily at the same frequency as the oscillated motion, however small periodic oscillations are superimposed on this low frequency motion; these small fluctuations are induced by vortex shedding and flow reversal. Lift forces F_z will also be generated due to the presence of the submerged core circular cylinder making cross-flow oscillations possible. The lift force oscillates at its fundamental lift frequency. Besides surge and heave, in addition, sway, pitch, roll and yaw motions are also presence. In order to calculate the energy generated only the low frequency in-line force will be considered.
The structural equations of motion for the dynamical behaviour of the WavePiston system with n degrees of freedom are

$$M_s\ddot{x}_{s\,i} + C_s\dot{x}_{s\,i} + K_s\,x_{s\,i} = F_i(U,\dot{U},\dot{x}_s,\ddot{x}_s)\,, \quad (5)$$

where M_s, C_s and K_s denote the mass, damping and stiffness matrices, and x_{si} and F_i are the displacements and the fluid forces pr unit span, respectively. The subscript $i = (x, y, z, \alpha, \beta, \theta)$ denotes the horizontal, vertical and rotational directions. The right-hand side of Eqn. (5) contains the resultants of the total wave forces.

Assuming the optimum conditions in which the incident wave is (1) perpendicular to the length of the twin plates and (2) the primary wave forces is due to surge motion only, then the assumed uncoupled dynamical behaviour of the plate of the WavePiston system reduces to a single motion system,

$$m_p\ddot{x}_p + c_p\dot{x}_p + k_p\,x_p = F_x(U,\dot{U},\dot{x}_p,\ddot{x}_p)\,, \quad (6)$$

where m_p, c_p and k_p denote the mass, damping and stiffness parameter of a single thin plate with dimension, height h_p, length l_p and thickness t_p. The plate dimensions included in the present study is listed in Table 2. The mass and first natural frequency, f_p^{11}, are based on steel properties. The natural frequency of the plate decreases with the increase of plate size approaching the dominating frequency of the incident waves.

Table 2: *Plate data (* after von Bülow* et al. *(2009)).*

Parameter	Plate #A	Plate #B
h_p [m]	1.5	2
l_p [m]	4	7
t_p [mm]	10	10
m_p [kg]	471	1099
k_p [kN/m]	67.1	16.7
f_p^{11} [Hz]	1.9	0.6
l_p/h_p [-]	2.7	3.5

3.6 Wave energy

The efficiency of the WavePiston system may be evaluated through the capture width ratio:

$$C_{eff} = \frac{P_{PTO}}{P_i\, l_p}\,, \tag{7}$$

where P_{PTO} is the power absorbed by the device and P_i is the incident wave power per unit width. The length of the plate, l_p.

The converted power, P_{won}, will be a function of several parameters. First and foremost the pressure of water in the core circular cylinder.
The power generated by the device can be defined as:

$$P_{PTO}(t) = F_x(t)\dot{x}(t) \tag{8}$$

The maximum powern that can be extracted from an incident propagating wave per unit width along the wave front is defined as:

$$P_i(t) = \int_{-h_0}^{\eta} \left(p + \rho g z + \frac{1}{2}\rho(u^2 + w^2)\right) u dz \tag{9}$$

p is the horizontal pressure, $\rho g z$ the potential energy and $\frac{1}{2}\rho(u^2 + w^2)$ the kinetic energy.

3.6.1 Small amplitude waves

Using small amplitude wave theory Eqn. (9) reduces to:

$$P_i(t) = \int_{-h_0}^{0} (p + \rho g z)\, u dz \tag{10}$$

The horizontal velocity and the term $(p + \rho g z)$ is always in phase so Eqn. (10) is always positiv. Taking the mean of Eqn. (10) gives:

$$P_i = E\, c_g\,, \tag{11}$$

where E is the total energy of the surface, when using small amplitude wave theory is defined as:

$$E = E_k + E_p = \frac{1}{4}\rho\, g\, \eta^2 + \frac{1}{4}\rho\, g\, \eta^2\,, \tag{12}$$

where E_k and E_p are the kinetic and potential energy, respectively. The surface elevation is η.

$$P_i^{deep} = \frac{\rho g^2 H_s^2 T_e}{64\pi} \quad \text{and} \quad P_i^{shallow} = \frac{\rho g^2 H_s^2 T_e}{32\pi}\,, \tag{13}$$

and for intermediate depth:

$$P_i^{int} = \frac{\rho g^2 H_s^2 L}{32\,T_e}\left(1 + \frac{4\pi h_0}{L}\,\frac{1}{sinh(\frac{4\pi h_0}{L})}\right), \tag{14}$$

where T_e is named the energy period accounting for the dominant power oscillations given by the peak in the power spectrum (for many seas $T_e \approx 1.12T_z$).

3.6.2 Large amplitude waves

For large amplitude waves linear wave theory can not be used. It is therefore necessary to calculate the incident wave power from Eqs. (9). Using stream function theory the pressure is defined as:

$$P_i(t,z) = \rho(R - gd - gz) - \frac{1}{2}\left(u^2 + w^2\right) \tag{15}$$

where R is the Bernoulli constant:

$$R(t) = \frac{1}{2}\left(u^2 + (-w)^2\right) + g\eta \quad \text{at} \quad z = \eta \tag{16}$$

4 Analytical predictions

The free-surface problem is idealized as wave propagating towards shore in a direction acting perpendicular to a thin plate, as shown in Fig. 4.

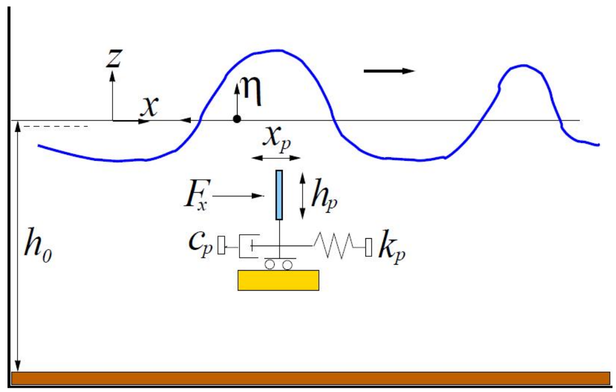

Figure 4: *Problem set-up. Wave past a thin plate.*

The solution to the wave-plate interaction as described by Eqn. (6) is solved by assuming that spacing between the thin plates are large and therefore no interaction of flow between the plates is assumed. The flow properties in Eqn. (6) is taken at the top of the plate. A center finite difference scheme is used. The displacement of the two plates for case #1 and case #2 is seen in Fig. 5.

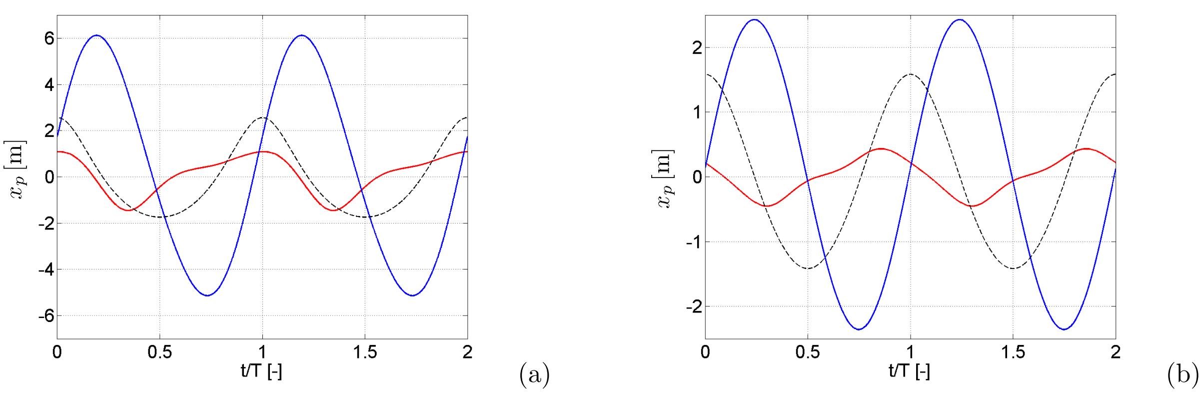

Figure 5: *Plate oscillations as a function of time with reference to the incident wave cf. table 1. Figure (a): Case #1, figure (b): Case #2.* *Plate A,* *Plate B,* $^{--}$*Surface elevation* η.

For case #1 which is the most non-linear case plate A has almost two crest. Why it is so is not clear but it indicates that experiments are necessary in order to understand the plate oscillations properly. However, from the solution it is clear that the problem is very complex because the velocity and acceleration of the plate is not in phase, shown in Fig. 6-7. The force which depends on the velocity and acceleration of both the flow and the plate and the displacement of the plate are therefore not in phase either. The total force for case #1 and case #2 are shown in figure Fig. 8. In Fig. 9-10 the force contributions are shown for case #1 and #2, respectively.
In order to get an idea of the quantity of power which the plate can generate the power $P_{PTO}(t)$ is time averaged over one wave period and compared with the corresponding time averaged wave power:

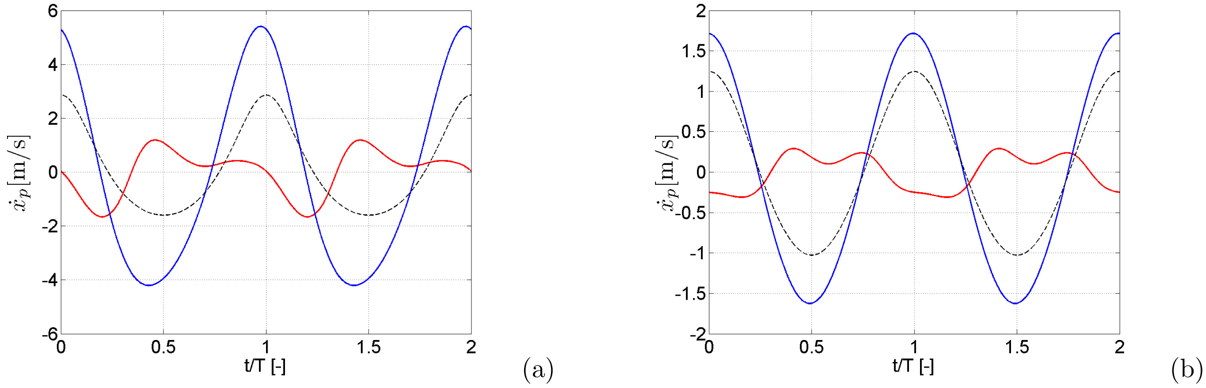

Figure 6: *The plate velocity. Figure (a): Case #1, figure (b): Case #2.* — *Plate A,* — *Plate B,* -- *Horizontal surface velocity, u.*

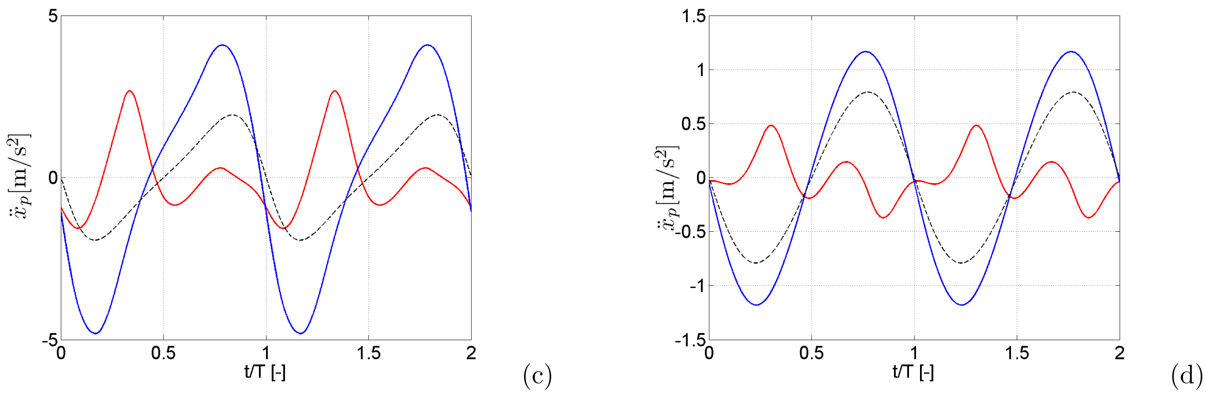

Figure 7: *The plate acceleration. Figure (a): Case #1, figure (b): Case #2.* — *Plate A,* — *Plate B,* -- *Horizontal surface acceleration,* $\frac{du}{dt}$.

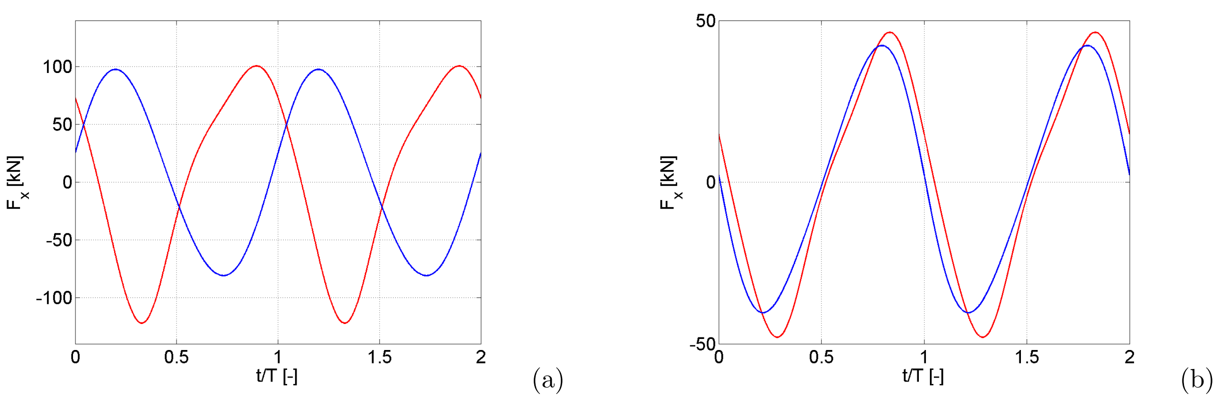

Figure 8: *The total force* F_x. *Figure (a): Case #1, figure (b): Case #2.* — *Plate A,* — *Plate B.*

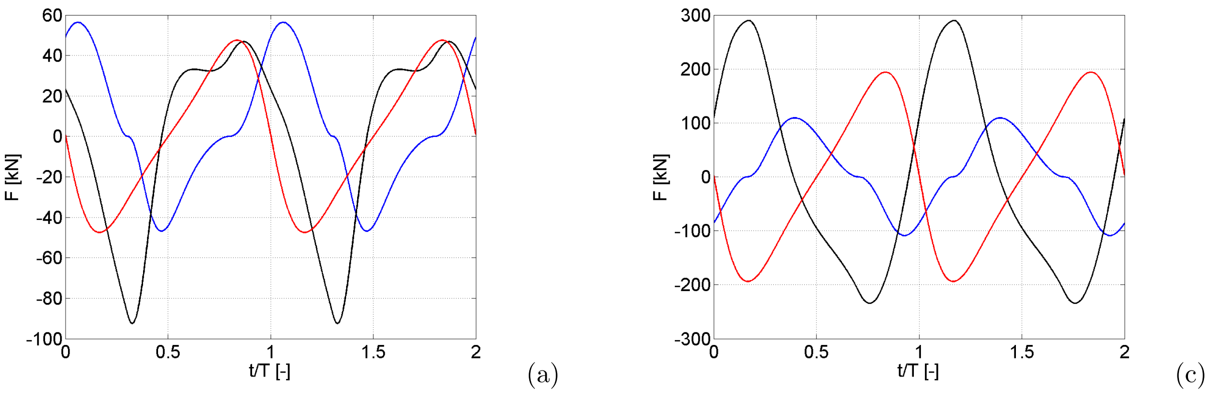

Figure 9: *The force contributions for case #1. Figure (a): Plate A, figure (b): Plate B. Force ID:* **−** *Froude-Krylov force,* **−** *Drag force,* **−** *Hydrodynamic mass force.*

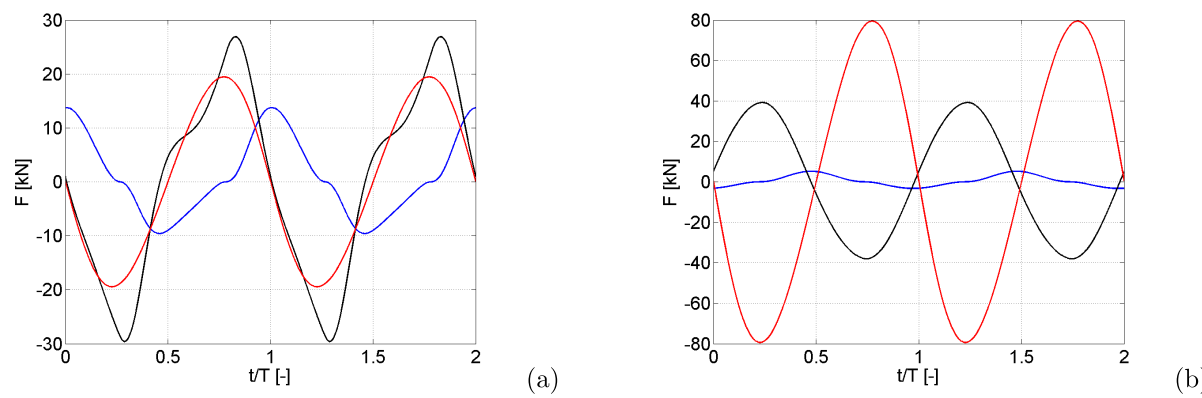

Figure 10: *The force contributions for case #2. Figure (a): Plate A, figure (b): Plate B. Force ID:* **−** *Froude-Krylov force,* **−** *Drag force,* **−** *Hydrodynamic mass force.*

$$P_{PTO} = \frac{1}{T}\int_0^T F_x(t)\dot{x}(t)dt \tag{17}$$

$$P_i = \frac{1}{T}\int_0^T \int_{-h}^{\eta} \rho(R - gd - gz) - \frac{1}{2}\left(u^2 + w^2\right) dzdt \tag{18}$$

The results are shown in table 3. P_{PTO} in the table is the power for a unit (twin plates). In order to compare P_{PTO} with the wave power this is multiplied with twice the plate length l_p.

Table 3: *WavePiston unit requirements in order to generate 5 MW based on time average power values for wave Case #1 and #2.*

Case ID	Plate ID	P_{PTO} [kW]	$P_i \times 2l_p$ [kW]	C_{eff} [%]	Units [-]	$\frac{A_p}{h_0 H_s}$ [%]
1	A	84	1825	5	60	20
1	B	274	3193	9	19	47
2	A	10	1180	0.8	490	13
2	B	43	2065	2	115	31

To evaluate how many units to form the WavePiston system based on the analytical estimates one may compare with a single large modern 5 MW wind turbine. The number of units of the WavePiston system is estimated so that it at least produce 5 MW, as shown in the Table 3. The last column in table 3 gives the ratio between the area of a unit (i.e. two plates), A_p and the water depth times the wave height, $h_0 H_s$. Together with the capture width ratio C_{eff} it is clear to see that the larger the plate is compared with $h_0 H_s$ the more power from the wave is absorbed by the system. The unit with 1.5×4 m (plate A) requires 60 units with wave case 1 to reach 5 MW while the unit 2×7 m (plate B) requires 19 units. This means that the assumption of no flow interaction between plates is not realistic as in this case the plates have to be spaced less than than the wave length to have practical dimensions of the WavePiston system. Then flow around one plate will affected due to the presence of the nearby plates. The transmitted wave is affected by the presence of the plates and the reflection and scattering of the waves and must be taken into account, as reported by Stiassnie (1986).

5 Further analysis

A numerical analysis has been made in order to predict the power which a plate under the influence of an oscillating wave can generate.
An equation of motion has been solved numerically in order to calculate the displacement, velocity and acceleration of the plate and the corresponding force acting on the plate. From this the power has been calculated and compared with the power generated by the wave.

The analysis has clearly shown that the WavePiston system is very complex and that further investigations are necessary. NIRAS recommend the following investigations:

- A conceptual study should be made in order to understand how the system will act under the influence of waves and whether the system can generate any power. This can be performed by small model setup.
- A study in order to optimize the system which includes both experiments and numerical analysis. It has to be investigated how the WavePiston will act under as realistic conditions as possible and whether the structure can resist the forces and torsion it is exposed to without breaking. It has to be investigated further how the WavePiston should be constructed so that it generates as much power as possible.
- The most optimum location for the WavePiston system must also depend on further investigations. The investigations should focus on the ratio between the design wave and the the operational wave and should be as small as possible. The wave field at he location should be as even as possible so that a constant amount of power is produced.

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